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Mathematical analysis of multi layers optic fiber models

Yuliya Kudrych[1], Kateryna Buryachenko[2]

[1]Vasyl' Stus Donetsk National University,
Research Department, Vinnytsia, Ukraine
ju.kudrych@donnu.edu.ua
https://orcid.org/0000-0002-1079-2852

[2]Vasyl' Stus Donetsk National University,
Department of Applied Mathematics and Cybersecurity, Vinnytsia, Ukraine
k.buriachenko@donnu.edu.ua
https://orcid.org/0000-0002-3363-2229

Abstract. This work is devoted to the development of qualitative methods for the study of nonlinear heterogeneous structures, models of which are elliptic equations, which describe complex nonlinear processes in heterogeneous media. They may also include the structures, consisting of several parts (phases or layers): multiphase solid and liquid materials; optic fiber and optic cable layers, anisotropic medium, etc. Relevance of the chosen direction is due to the fact that many processes in heterogeneous environments under conditions of high temperatures, heavy loads and significant deformations are described using nonlinear differential equations with discontinuous (singular) data (coefficients, right-hand side, boundary and initial conditions, etc.). At the same time, the concept of weak solutions that meet the modern needs of mathematical physics arose. Nonlinear differential equations have a complex structure, which actually makes them impossible to study by finding solutions in an explicit form. Therefore, the development of qualitative methods for their investigations becomes an extremely important tool. This paper considers mathematical models of multilayer optic fiber and cable, which consist of 3 and 5 different materials respectively with different properties. Using potential theory, the behavior of a weak solution of this equation at a fixed point is estimated and analyzed by the value of the nonlinear Wolff potential from the right hand side. We study pointwise properties that play a key role in the further study: expansion of positivity Harnack's inequalities, regularities and others. The paper discusses also the application of the obtained theoretical results for the problem of modeling and analyzing of optic fiber and optic cable modern technologies.

Keywords: multiphase (double phase) equations, optic fiber models, $(p(x), q(x))-$ Laplace, Wolff potential, weak solution, pointwise estimates.

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1. Introduction

We focus here on the development of qualitative methods of nonlinear analysis for the study of double-phase elliptic equations with variable exponents and their applications in modern optic technologies. The active development of the problem under consideration is evidenced by numerous high citing publications during the past 2-3 years in leading journals: V. Bögelein, F. Duzaar, P. Marcellini, C. Scheven [2], V. Bögelein, M.Strunk [4], C. De Filippis, G. Mingione [5] and others. The double-phase elliptic equations of the divergence form were studied in first in the papers [9, 10] as models of strictly anisotropic materials and for the description of Lavrent'ev phenomenon. Hölder continuity and Harnack's inequality for bounded solutions to the homogeneous equation were obtained in [1], [6] under the same conditions, which we have herein .

These works were fundamental for further studies of the existence and regularity of solutions of various types of problems for such equations. The novelty of the results of this paper is the development of new functional methods of nonlinear analysis for the study of new actual problems, the mathematical models of which are double-phase elliptic equations with variable exponents. We introduce here the new potential estimates for the weak nonnegative solutions via nonlinear Wolff potential of the right hand side $f \in L^1$ of the equation and discuss their applications in the modeling of optic fiber devices.

The considering class of double-phase equations serves as mathematical model of media including structures which consist of several parts (phases or layers): multiphase solid and liquid materials; porous, anisotropic media; optic fiber layers, optic cable layers, light diodes, semiconductors devices, etc. The relevance of the chosen direction is due to the fact that many processes in heterogeneous environments under conditions of high temperature, heavy loads and significant deformations are described by using similar equations and with discontinuous (singular) data (coefficients, right-hand side, boundary and initial conditions, etc.). In our case this is a right hand side $f \in L^1$. At the same time, the concept of weak solutions is widely used, which meets the modern needs of mathematical physics. Nonlinear differential equations have a complex structure, which actually makes it impossible to study them by finding solutions in an explicit form. Therefore, the development of qualitative methods of analysis becomes an extremely important tool. In the present manuscript we obtain new pointwise estimates for the weak nonnegative solution via nonlinear Wolff potential from the right hand side of elliptic equations with non-standard growth conditions, $(p, q)-$ double-phase equations, with variable exponents: $p(x), q(x)$. Obtained in the manuscript new pointwise properties for the weak solutions via nonlinear Wolff potential from the right-hand side $f \in L^1$ will explore fundamental qualitative properties that play a key role in further studying the behavior of solutions: boundedness, expansion of positivity, Hölder continuity, and Harnack's inequalities.

The main results of the current paper are expansions of the works [4] and [8] for the case of double-phase elliptic equations with variable exponents $p(x)$, $q(x)$.

We consider also mathematical models of multilayer optic fiber and multilayer optic cable, which consist of 3 and 5 different materials with different properties and discuss the application of the obtained theoretical results for the problem of modeling and analyzing of optic fiber and optic cable modern technologies.

2. Statement of the problem

In a bounded domain $\Omega \subset \mathbb{R}^n$, $n \geq 2$ we consider a double-phase elliptic equation with variable exponents:

$$-\mathrm{div}\left[(|\nabla u|^{p(x)-2} + a(x)|\nabla u|^{q(x)-2})\nabla u\right] = f(x) \geq 0, \tag{1}$$

$$-\mathrm{div} A(x,\, \nabla u) = f(x) \geq 0, \tag{2}$$

where $f(x) \in L^1(\Omega)$. We assume that the function $A(x,\, \xi) = |\xi|^{p(x)-1} + a(x)|\xi|^{q(x)-1} : \Omega \times \mathbb{R}^n \to \mathbb{R}^n$ satisfies the conditions

1) $A(x,\, \xi)$ satisfies the Carathéodory condition,
2) $A(x, \xi)\xi \geq \mu_1(|\xi|^{p(x)} + a(x)|\xi|^{q(x)})$,
3) $|A(x, \xi)| \leq \mu_2(|\xi|^{p(x)-1} + a(x)|\xi|^{q(x)-1})$,

with some constants $\mu_1,\, \mu_2 > 0$.

We also assume that

$$0 \leq a(x) \in C^{0,\alpha}(\Omega), \quad \alpha \in (0,\, 1].$$

Let $\mathcal{M}$ be a set of all measurable functions, $p(x), q(x) : \Omega \to (1, \infty)$. For $p(x), q(x) \in \mathcal{M}$, we set:

$$p_- := \mathrm{essinf}_{x\in\Omega} p(x),\ q_- = \mathrm{essinf}_{x\in\Omega} q(x), p_+ := \mathrm{esssup}_{x\in\Omega} p(x),\ q_+ = \mathrm{esssup}_{x\in\Omega} q(x).$$

We assume the following for the powers of nonlinearity:

$$1 < p_- \leq p_+ \leq q_- \leq q_+ \leq \min\left(p_- + \alpha,\, \frac{n(p_- - 1)}{n - p_-}\right), \quad q_+ < n. \tag{3}$$

Let us introduce the necessary definitions.

Definition 1. Let $G(x, t) = t(t^{p(x)-1} + a(x)t^{q(x)-1})$. Then $W^{1,G}(\Omega)$ denotes the class of functions u that are weakly differentiable in Ω and satisfy the condition

$$\int_\Omega G(a(x),\, |\nabla u|)\, dx < \infty.$$

Definition 2. We say that u is a weak solution to Eq. (2), if $u \in W^{1,G}(\Omega)$ and it satisfies the integral identity

$$\int_\Omega A(x,\, \nabla u)\nabla\varphi\, dx = \int_\Omega f\, \varphi\, dx, \tag{4}$$

for all $\varphi \in \overset{0}{W}{}^{1,G}(\Omega)$.

In the case of Eq.(1) condition (4) takes the form:

$$\int_\Omega \left(|\nabla u|^{p(x)-1} + a(x)|\nabla u|^{q(x)-1}\right) \nabla\varphi\, dx = \int_\Omega f\, \varphi\, dx. \tag{5}$$

We will prove the pointwise estimates for a nonnegative weak solution to the double-phase equation (1) in terms of the nonlinear Wolff potentials:

$$W^f_{1,p(x)}(x_0,\, R) = \sum_{j=0}^{\infty} \left(\rho_j^{p(x)-n} \int\limits_{B_{\rho_j}(x_0)} f\, dx \right)^{\frac{1}{p(x)-1}},\ \rho_j = \frac{R}{2^j},\ j = 0,\, 1,\, ...$$

$$W^f_{1,q(x)}(x_0,\, R) = \sum_{j=0}^{\infty} \left(\rho_j^{q(x)-n} \int\limits_{B_{\rho_j}(x_0)} f\, dx \right)^{\frac{1}{q(x)-1}},\ \rho_j = \frac{R}{2^j},\ j = 0,\, 1,\, ...,$$

under assumption that the series in the above formulae are convergent, i.e. the Wolff potentials are finite.

Let us note that double-phase elliptic equations of the divergence form were studied in first in the papers [9, 10] as models of strictly anisotropic materials and for the description of Lavrent'ev phenomenon. Hölder continuity and Harnack inequality for bounded solutions to the homogeneous equation (1) (with function $f \equiv 0$) were obtained in [1], [6] under conditions (3).

3. Main result

The main result of the present work is the following theorem.

Theorem 3. *Let $u \in W^{1,G}(\Omega) \cap L^\infty$ be a nonnegative weak solution to Eq. (1). Let conditions (3) be satisfied and let $[a]_{C^{0,\alpha}(\Omega)} := \sup\limits_{x,y\in\Omega,\, x\neq y} \frac{|a(x)-a(y)|}{|x-y|^\alpha}$. Assume also that the point $x_0 \in \Omega$ is such that $B_{4\rho}(x_0) \subset \Omega$. Then there exist constants $c_1,\, c_2 > 0$ depending only on p_-, q_+, n, $[a]_{C^{0,\alpha}(\Omega)}$ and $||u||^{q_+-p_-}_{L^\infty(\Omega)}$ such that, under condition $a(x_0) = 0$ the following estimate holds:*

$$c_1 W^f_{1,p_-}(x_0, \rho) \le u(x_0) \le c_2 \inf_{B_\rho(x_0)} u + c_2 W^f_{1,p_-}(x_0, 2\rho). \tag{6}$$

If $a(x_0) > 0$ and $\rho_0^\alpha = \frac{a(x_0)}{4[a]_{C^{0,\alpha}(\Omega)}} \ge \rho^\alpha$, then there exist constants $c_3,\, c_4 > 0$ depending on p_-, q_+, n, $[a]_{C^{0,\alpha}(\Omega)}$, $||u||^{q_+-p_-}_{L^\infty(\Omega)}$ and $a(x_0)$ such that the following estimate

$$c_3 W^f_{1,q_+}(x_0, \rho) \le \rho + u(x_0) \le 3\rho + c_4 \inf_{B_\rho(x_0)} u + c_4 W^f_{1,q_+}(x_0, 2\rho) \tag{7}$$

holds.

Under conditions $a(x_0) > 0$ and $\rho_0 < \rho$ will be true the estimate

$$\begin{aligned} c_3 W^f_{1,q_+}(x_0, \rho) + c_3 (W^f_{1,p_-}(x_0, \rho) - W^f_{1,p_-}(x_0, \rho_0)) \le \rho + u(x_0) \le \\ \le 3\rho + c_4 \inf_{B_\rho(x_0)} u + c_4 W^f_{1,q_+}(x_0, 2\rho) + c_4 (W^f_{1,p_-}(x_0, 2\rho) - W^f_{1,p_-}(x_0, 2\rho_0)). \end{aligned} \tag{8}$$

Proof. The result of this theorem will follow from the analogue result, proved in [4] for the double-phase equation with constant powers of nonlinearity p, q :

$$-\mathrm{div}\left[(|\nabla u|^{p-2} + a(x)|\nabla u|^{q-2})\nabla u\right] = f(x) \ge 0, \tag{9}$$

with

$$1 < p \le q \le \min\left(p+\alpha,\ \frac{n(p-1)}{n-p}\right), \quad q < n. \tag{10}$$

Taking into account our conditions (3), we have the statement of our theorem as a consequence of the analogous result for Eq.(9), see [4].

Remark 4. In the case $a(x_0) = 0$ inequality (6) yields the known result of Kilpeläinen and Malý [7], where there were obtained the pointwise estimates of solutions to a quasilinear elliptic equation with the p-Laplace and measure μ on the right-hand side with the help of the nonlinear Wolff potential $W^{\mu}_{\beta,\,p_-}(x_0,\,R)$:

$$W^{\mu}_{\beta,\,p_-}(x_0,\,R) := \sum_{j=0}^{\infty}\left(\frac{\mu(B_{\rho_j}(x_0))}{\rho_j^{n-\beta p_-}}\right)^{\frac{1}{p_- -1}},\ \rho_j = \frac{R}{2^j},\ j = 0,\,1,\,2,\ldots \tag{11}$$

4. Applications to the multi layers optic fiber models.

Consider the multilayer optic fiber model, described by the exponents:

$$p(x) = \begin{cases} p_1 & x \in \Omega_1, \\ p_2 & x \in \Omega_2, \\ p_3 & x \in \Omega_3, \\ \cdots & \\ p_n & x \in \Omega_n; \end{cases} \quad q(x) = \begin{cases} q_1 & x \in \Omega_1, \\ q_2 & x \in \Omega_2, \\ q_3 & x \in \Omega_3, \\ \cdots & \\ q_n & x \in \Omega_n, \end{cases} \tag{12}$$

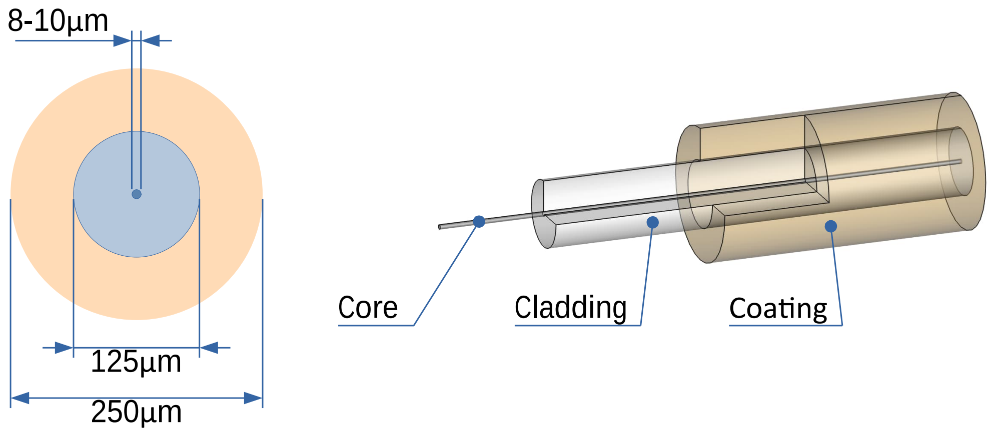
with the constant p_i, q_i, $i = 1, ..., n$, depending on the number of layers in the optic fiber. In this case of discrete-valued $p(x)$ and $q(x)$, we stand

$$p_- = \min_{i=1,..,n} p_i,\ \ q_- = \min_{i=1,..,n} q_i,\ \ p_+ = \max_{i=1,..,n} p_i,\ \ q_+ = \max_{i=1,...,n} q_i.$$

As usual, optic cable is a carefully designed multilayer designed to protect sensitive optic fiber and ensure its optimal performance under various environmental conditions and mechanical loads. The main components working in interaction include:

- **Core**: Ω_1, The innermost part of the cable, serving as a way to transmit light. It is usually made of high-purity glass or, less commonly for single-mode fibers, of plastics;
- **Cladding**: Ω_2, The optical layer immediately surrounding the core. Its material composition is chosen to have a lower refractive index than the core, which is a critical property that contributes to the complete internal reflection and retention of light in the core;
- **Buffer layer**: Ω_3, A protective coating applied directly over the shell. This layer provides substantial physical protection to the fiber, protecting it from minor abrasive damage, impact and exposure to environmental elements;
- **Power elements**: Ω_4, These components are strategically integrated into the cable structure to provide tensile strength and mechanical reinforcement, protecting the optic fiber from stretching, bending, and crushing;
- **Coating**: Ω_5, The outer protective layer of the cable. This layer provides comprehensive protection against moisture, ultraviolet radiation, chemicals and mechanical damage, and often serves to identify the cable.

For example, in the case of a optic fiber it is a carefully designed by three multilayer designed to protect sensitive optic fiber and ensure its optimal performance under various environmental conditions. The main components working in interaction include only two parts (core and cladding). Please, see the following single-mode optic fiber:



Taking into account that optic fiber consists of different kinds of materials, it is natural, that the value of powers of nonlinearity, $p(x), q(x)$ take a different values (12) on each of layers $\Omega_i,\ i = 1, ..., 5$.

Thus, the core usually consists of ultrapure quartz (SiO2). To achieve the required higher refractive index relative to the shell, quartz is precisely doped with elements such as germanium dioxide (GeO2) [11]. The ultra-purity of quartz glass is paramount to minimize light absorption and scattering, thereby ensuring high transmission efficiency. The cladding layer is usually made of pure quartz or fluorine-doped quartz, which effectively reduces its refractive index compared to the germanium-doped core [11]. Acrylate polymers or polyimides are used for buffer. These materials are chosen because of their adhesion to glass and protective properties. Aramid threads (e.g. Kevlar, Twaron) are wide use materials for power elements.The cable outer sheath is the most visible protective layer of the optical cable. Its main role is to protect the internal components from environmental factors, mechanical damage and fire dangers. The choice of sheath material is very application-dependent, balancing performance, cost and safety requirements, for instance: polyvinyl chloride, polyethylene, polyurethane and others [12].

For (12) the result of Theorem 3 can be applied. So, we can estimate the pointwise value of the solution $u(x_0)$ via nonlinear Wolff potential of the right-hand side $f \in L^1(\Omega)$, depending on the point $x_0 \in \Omega = \Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5$.

Conclusions. The paper discusses a mathematical model of multilayer optic fiber and optic cable, which consists of 3 and 5 different materials respectively with different properties. Using potential theory, the behavior of a weak solution of this equation at a fixed point from the value of the nonlinear Wolff potential from the right side is analyzed. This result complements the work of one of the author [4] in the case of variable powers $p(x), q(x)$ of nonlinearity. Additionally, the paper discusses the application of the obtained results for the problem of modeling and analyzing of optic fiber modern technologies.

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Математичний аналіз багатошарових оптоволоконних моделей

Юлія Кудрич, Катерина Буряченко

Анотація. Робота присвячена розробці якісних методів дослідження нелінійних гетерогенних структур, моделями яких є еліптичні рівняння, що описують складні нелінійні процеси в неоднорідних (гетерогенних) середовищах. Ці структури складаються з декількох частин (фаз або прошарків): багатофазних твердих і рідких матеріалів; оптичних волокон і оптичних кабелів, анізотропних середовищ, тощо. Актуальність обраного напрямку обумовлена тим, що багато процесів в неоднорідних середовищах в умовах високих температур, великих навантажень і значних деформацій описуються за допомогою нелінійних диференціальних рівнянь з розривними (сингулярними) даними (коефіцієнти, права сторона, граничні та початкові умови тощо). При цьому виникає концепція слабких розв'язків, які відповідають сучасним потребам математичної фізики. Нелінійні диференціальні рівняння мають складну структуру, що фактично робить неможливим їх вивчення та аналіз шляхом пошуку розв'язку у явному вигляді. Тому, розробка саме якісних методів дослідження стає надзвичайно важливим інструментом для їх подальшого вивчення. В роботі розглянуто математичні моделі багатошарового оптичного волокна та оптичного кабелю, які складаються з 3 та 5 різних матеріалів відповідно з різними властивостями. Використовуючи теорію нелінійних потенціалів, оцінюється та аналізується поведінка слабкого розв'язку цього рівняння в фіксованій точці через значення нелінійного потенціалу Вольфа від правої частини рівняння. Вивчаються поточкові властивості, які відіграють ключову роль у подальшому дослідженні та вивченні таких властивойстей розв'язків, як, наприклад, розширення за позитивністю, нерівність Гарнака, та ін. В статті розглянуто також застосування отриманих теоретичних результатів для вирішення проблеми моделювання та аналізу сучасних оптоволоконних технологій.

Ключові слова: багатофазні рівняння, оптоволоконні моделі, (p(x), q(x)) рівняння Лапласа, потенціал Вольфа, слабкий розв'язок, поточкові оцінки.

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Про авторів / About the authors

Юлія Кудрич, молодший науковий співробітник науково-дослідної частини Донецького національного університету імені Василя Стуса, вул. 600-річчя, 21, м. Вінниця, 21021, Україна;

Yulia Kudrych, Junior researcher, Research Department, Vasyl' Stus Donetsk National University, 21 600-richchia Str., Vinnytsia 21021, Ukraine;

Катерина Буряченко, кандидат фізико-математичних наук, доцент, кафедра прикладної математики та кібербезпеки, Донецький національний університет імені Василя Стуса, вул. 600 річчя, 21, м. Вінниця, 21021, Україна;

Kateryna Buryachenko, Candidate of Science in Physics and Mathematics, Associate Professor, Department of Applied Mathematics and Cybersecurity, Vasyl' Stus Donetsk National University, 21 600-richchia Str., Vinnytsia 21021, Ukraine.

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